

# Correlating Friction Velocity in Turbulent Boundary Layers Subjected to Freestream Turbulence

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A turbulent boundary layer was subjected to grid-generated freestream turbulence to study the effects of turbulent length scale and intensity on skin friction. The study emphasized the importance of the turbulent-to-viscous length scale ratio and focused on correlating skin friction without the use of conventional boundary-layer Reynolds numbers. The experimental data were characterized by small-scale freestream turbulence above boundary layers with momentum-thickness Reynolds numbers ranging from 225 to 2700. Freestream turbulence intensities varied from 0.1 to 8.0%. The turbulent-to-viscous length scale ratios spanned from 100 to 1000, and ratios of freestream turbulent length scale to boundary-layer momentum thickness ranged from 4 to 32. A new boundary-layer model was used to establish a skin-friction correlation that does not require knowledge of the momentum-thickness Reynolds number. The correlation was evaluated using the new small-scale data and other data from the literature. When only freestream information (length scale, mean velocity, and turbulence intensity) was employed, the correlation provided an estimate of the friction velocity to within 8% (20:1 odds). The correlation was evaluated using data with momentum-thickness Reynolds numbers up to 6300, freestream intensities as large as 29%, turbulent-to-viscous length scales reaching 11,000, and length scale-to-boundary-layer momentum-thickness values as high as 190.

## Nomenclature

$C_f$	= skin-friction coefficient, $2\tau_w/\rho u_\infty^2$ or $2u_\tau^2/u_\infty^2$
$L$	= turbulent length scale
$l$	= sensor wire length
$Re$	= Reynolds number
$Tu$	= turbulence intensity, $u'/\bar{u}$
$u$	= streamwise velocity
$u_\tau$	= boundary-layer friction velocity, $\sqrt{(\tau_w/\rho)}$
$v$	= velocity in direction normal to surface
$x$	= streamwise coordinate
$y$	= coordinate in direction normal to the surface
$\delta$	= boundary-layer thickness
$\theta$	= boundary-layer momentum thickness
$\nu$	= kinematic viscosity
$\sigma$	= standard deviation
$\tau_w$	= wall shear stress

## Subscripts

$e$	= energy based
$w$	= wall
$0$	= at the same $Re_\theta$ in a $Tu_\infty = 0$ flow
$15$	= located at $y^+ = 15$
$\infty$	= freestream

## Superscripts

$-$	= Reynolds averaged mean
$'$	= fluctuating (rms value)
$+$	= nondimensionalized using wall variables

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## Introduction

DESIGN engineers daily encounter difficulty in accurately predicting skin friction for turbulent boundary layers subjected to freestream turbulence (FST). Existing correlations and numerical techniques are still limited by a priori information requirements and the raw computational complexity of flows with moderate to high Reynolds numbers. To aid the design engineer, this work establishes a friction-velocity correlation that uses only freestream information. The hope is to develop a useful correlation that relies on only a few readily estimated input parameters.

We begin with a summary of earlier FST work that developed techniques to correlate skin friction. In the context of the earlier works, we present the objectives of the current study. The experimental methods used to gather new small-scale data are briefly described. Baseline experimental results are displayed to validate the investigative techniques. Data for the elevated FST cases are given; we showcase the very small freestream length scales achieved. With use of new data, earlier correlation methods are evaluated. Following the evaluation, we develop a new friction-velocity correlation that does not need momentum-thickness Reynolds number as input. From the results, we draw conclusions that address the stated experimental objectives.

## Literature Review

Simonich and Bradshaw<sup>1</sup> initiated one of the modern approaches to evaluating FST effects on flat-plate boundary-layer skin friction. They examined the effects of varying  $Tu_\infty$  and  $L/\delta$ . (The length  $L$  roughly scales with the size of the energy-containing turbulent eddies.) They showed that  $C_f$  increased as  $Tu_\infty$  increased and  $L/\delta$  decreased. Their work also extended the Kader and Yaglom<sup>2</sup> scaling analysis. Simonich and Bradshaw<sup>1</sup> noted the viscous length scale,  $\nu/u_\tau$ , was the compatible reference length for the similarity analysis. However, they used  $\theta$  because it was more convenient to compare boundary layers at similar Reynolds number  $Re_\theta$  values. The use of  $Re_\theta$  was an improvement over  $Re_x$  comparisons; however, it diluted the importance of  $L/(\nu/u_\tau)$  similarity.

Hancock and Bradshaw<sup>3</sup> conducted the first systematic evaluation of the effect of  $L_\infty$  on boundary-layer skin friction. They presented

the enhancement proportionality,

$$\Delta C_f / C_{f0} \propto Tu_\infty / (2 + L/\delta) \quad (1)$$

The grouping  $Tu_\infty / (2 + L/\delta)$  is termed the Hancock–Bradshaw (HB) parameter.

Blair<sup>4,5</sup> examined  $C_f$  using the HB parameter. With low  $Re_\theta$  data, Blair added a “damping-term,”

$$HB_{\text{mod}} = Tu_\infty / \{(2 + L/\delta)[1 + 3 \exp(-Re_\theta/400)]\} \quad (2)$$

He concluded that  $\Delta C_f / C_{f0}$  was proportional to  $HB_{\text{mod}}$ . Whereas others offered similar modifications (for example, Castro<sup>6</sup>), we use  $HB_{\text{mod}}$  given in Eq. (2) to represent this type of analysis.

In discussing  $Tu_\infty$  effects on boundary-layer structure, Hancock and Bradshaw<sup>7</sup> showed that wake strength and other integral parameters also correlated with HB.

Johnson and Johnston<sup>8</sup> recorded some of the smallest  $L/\delta$  to date; they used a passive biplanar grid in a water tunnel. Their  $Tu_\infty$  ranged from 4.7 to 7.5%; the  $L/\delta$  ratios varied from 0.27 to 0.4. For  $Re_\theta$  variation from 1150 to 1475, they showed  $\Delta C_f / C_{f0}$  of 15–18%.

Ames and Moffat<sup>9</sup> studied large-scale, high  $Tu_\infty$  effects. They used results from Hunt and Graham<sup>10</sup> to introduce correlations for  $C_f$  enhancement. They obtained

$$\Delta C_f / C_{f0} \propto Tu_\infty (\theta/L)^{1/3} (Re_\theta/1000)^{1/4} \quad (3)$$

where the right-hand side of Eq. (3) is called TLR<sub>m</sub>.

Sahm and Moffat<sup>11</sup> examined FST effects in flows with curvature; however, they also provided some flat-plate results. Using passive and jet grids, they examined flows with  $Tu_\infty$  as high as 29%. Their data suggested the freestream was not fully developed in the flat-plate region behind the grid. As they noted, the flat-plate boundary layer thinned as it progressed downstream.

Hollingsworth and Bourgoigne<sup>12</sup> examined momentum boundary layers growing under a fully turbulent shear layer. Despite stress-driven three dimensionality in the resultant flow, they found that two-dimensional scaling laws applied in the boundary layer near-wall and inner-core regions. These observations were affirmed in the heat transfer study of Maniam and Hollingsworth.<sup>13</sup>

Thole<sup>14</sup> and Thole and Bogard<sup>15,16</sup> studied  $Tu_\infty$  effects using wall-injection jets. Their work reaffirmed boundary-layer flux correlation with HB and TLR. From the heat-transfer work of Maciejewski and Moffat,<sup>17,18</sup> Thole<sup>14</sup> and Thole and Bogard<sup>15,16</sup> derived a skin-friction coefficient based on a near-wall fluctuating velocity,  $C'_f$ . They showed that  $C'_f$  varies with  $Tu_\infty$ ; they observed no  $L_\infty$  or  $Re$  dependence.

Bott and Bradshaw<sup>19,20</sup> used a passive grid in a wind tunnel with a rotating belt as the wall. When belt speed was varied, their  $Tu_\infty$  values [calculated as  $u'_\infty / (\bar{u}_\infty - \bar{u}_w)$ ] were larger than those in earlier passive-grid studies. Enhancements were presented in terms of  $HB_{\text{mod}}$  and the Castro<sup>6</sup> parameter.

Finally, three other studies are useful as contextual references. Camp and Shin<sup>21</sup> observed compressor-blade wakes coherently propagating into downstream blade passages; the wakes produced passage-flow  $L_\infty$  values as small as 4% of mean chord. Ames and Plesniak<sup>22</sup> (also Ames<sup>23</sup>) recorded  $L_\infty/\theta$  as low as 9.5 in an apparatus representing the first stator row after the exit of a combustor. Even though wakes from upstream blades were absent, scales in the blade passage were significantly smaller than those generated by the combustor. Blade-passage  $Tu_\infty$  values measured between 2 and 3%;  $Re_\theta$  varied from 2400 to 2600. Halstead et al.<sup>24</sup> provided extensive measurements from a rotating, multistage turbine test apparatus. Peak  $Tu_\infty$  values in downstream stages coincided with the passing of coherent wakes from upstream blades. The likely small-scaled wakes carried peak intensities of less than 5%. Suction-surface  $Re_\theta$  values ranged from 100 to 1000; the highest values were measured during engine startup conditions.

### Present Objectives

We have identified two existing methods to account for FST effects. The first uses FST characteristics to correlate  $\Delta C_f / C_{f0}$ . The

HB and TLR methods are both of this type. The second makes use of a near-wall skin-friction coefficient  $C'_f$  assumed to be only a function of  $Tu_\infty$ . The requisite  $Re_\theta$  or  $u'$  information restrict the predictive usefulness of either method.

Framed in this context, we adopt three objectives for the present work: 1) for  $Re_\theta$  and  $Tu_\infty$  representative of flows in aircraft turbines, investigate the effect of small-scale FST on  $C_f$ , 2) closely examine the possibility of length scale dependence in  $C'_f$ , and 3) for elevated  $Tu_\infty$ , correlate  $u_\tau / \bar{u}_\infty$  to the FST without reference to  $Re_\theta$ .

## Experimental Methods

Velocities were measured using hot-wire anemometers in flat-plate air boundary layers subjected to grid-generated turbulence. Both single- and two-wire probes were used to examine turbulence characteristics; however, the new skin-friction and velocity data used in developing the correlation are based on single-wire measurements only. The single-wire probe was 0.5 mm long with a 2.5- $\mu\text{m}$  diameter. The dimensionless sensor length  $l^+$  ranged from 3 to 15, and the sensor length never exceeded 4% of the measured integral length scale  $L_e$ . For measurements at  $y^+ = 15$ , the ratio formed by dividing  $y$  by the wire radius fell below 500 only twice (392 and 461). Near-wall conduction effects are expected to be negligible for ratios greater than 500 (Ref. 25); no corrections were made to the raw data to account for this phenomenon. The  $u'$  fluctuation wire-length correction of Maniam<sup>26</sup> was used for the  $y^+ = 15$  data only. (Profiles were not adjusted.) The correction was small (2–8%) and did not affect the overall results, particularly because the correlation development included many other data sets from studies using different measurement methods. Conventional measurement techniques were used in the current study and are reported elsewhere<sup>27–30</sup>; see the earlier works for additional details.

## Experimental Results

### Clean Freestream (CFS) Validation Data

Figure 1 shows  $u^+$  profiles measured at different  $Re_\theta$  and compares them with the near-wall and logarithmic profiles given by Kays and Crawford<sup>31</sup>:

$$u^+ = y^+ \quad \text{for} \quad y^+ < 5 \quad (4)$$

$$u^+ = 2.44 \ln(y^+) + 5.0 \quad \text{for} \quad 50 < y^+ < \text{wake} \quad (5)$$

The direct numerical simulation (DNS) results of Spalart<sup>32</sup> are also presented for comparison. Even the  $Re_\theta \cong 500$  profile still exhibits a small logarithmic region. Figure 2 shows profiles of  $u'^+$ . At similar  $Re_\theta$ , the data generally compare well with Spalart's results.

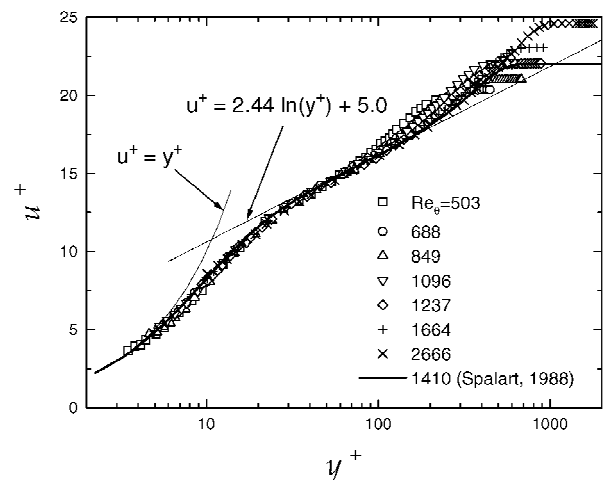


Fig. 1 CFS mean velocity profiles.

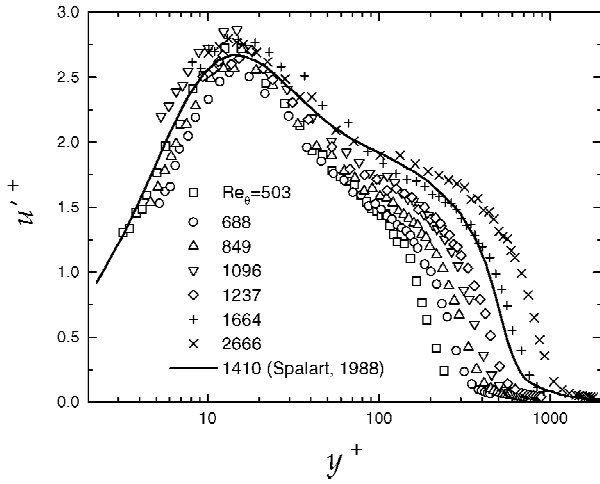


Fig. 2 CFS streamwise fluctuation profiles.

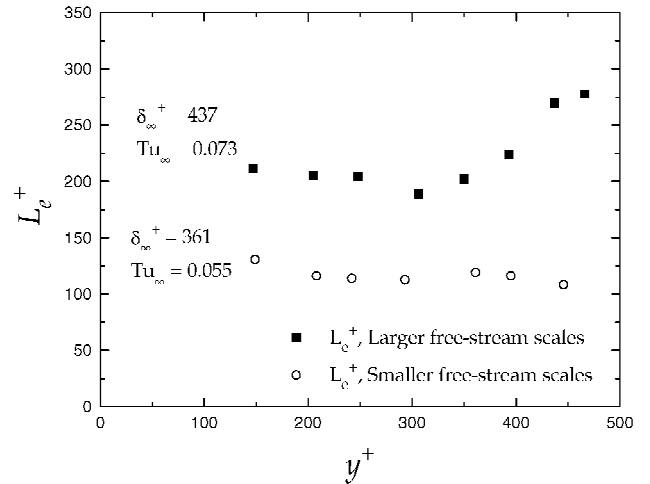


Fig. 4  $L_e^+$  length scale profiles.

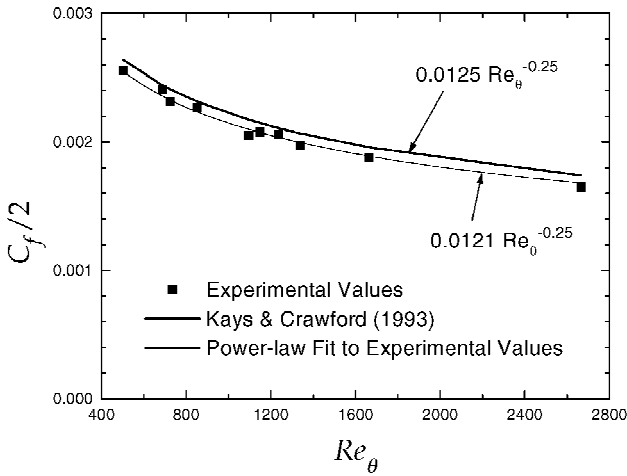


Fig. 3 CFS skin-friction coefficient vs  $Re_\theta$ .

Figure 3 shows  $C_f/2$  plotted vs  $Re_\theta$  and the correlation of Kays and Crawford,<sup>31</sup>

$$C_f/2 = 0.0125 Re_\theta^{-0.25} \quad (6)$$

The current values lie slightly below the correlation, but they are within the typical experimental uncertainty of  $\pm 5\%$  on  $C_f$  for clean freestream (CFS) flows.

#### Turbulent Length Scales

Most of the previous studies examined flows with  $L_{e,\infty}$  that are much larger than the turbulence scales in the inner region of the boundary layer and often much larger than the scales in the boundary-layer core. However, much of the current data represent flows with  $L_{e,\infty}$  smaller than scales in the boundary-layercore, and some profiles have  $L_{e,\infty}$  as small as scales in the log region. To illustrate, two boundary-layer profiles of  $L_e^+$  ( $L_e$  divided by  $\nu/u_\tau$ ) are shown in Fig. 4. In both cases, the outer region is seeded with scales smaller than those that would naturally occur from shear-produced turbulence; for both profiles,  $L_{e,\infty}$  is smaller than the mean-velocity-based boundary-layer thickness. However, for the filled symbols,  $L_e$  decreases from the freestream value as we traverse the core and approach the wall. The open symbols show a boundary layer with a more unusual scale distribution: The  $L_e$  introduced from the free-stream are as small as those in the log region.

Figure 5 shows a comparison of  $\theta/L_e$  values in the current work to previous studies. Length scales were measured at  $y^+ = 150$  for the CFS case. The current investigation examines  $L_{e,\infty}$  as small (relative to  $\theta$ ) as those obtained by Johnson and Johnston<sup>8</sup> and an order of magnitude smaller than many studies. In comparison to

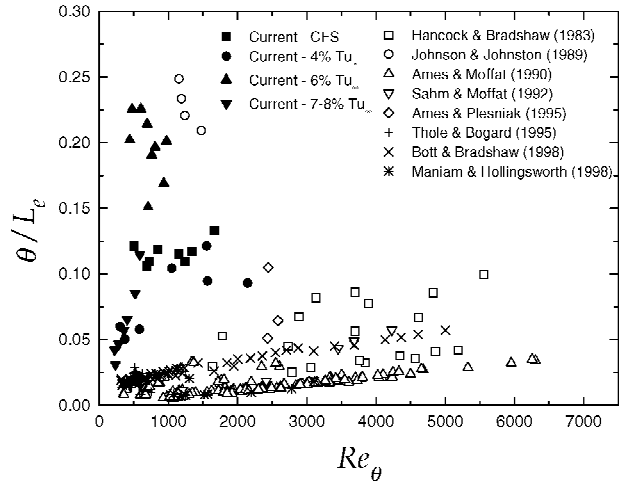


Fig. 5 Length scale ratios in current and previous studies ( $\theta/L_e$  vs  $Re_\theta$ ).

the viscous length scale,  $\nu/u_\tau$ , Fig. 6 shows that the current scales are the smallest examined. Figure 5 also exhibits the  $Tu_\infty$  and  $Re_\theta$  ranges of the current study. The ranges cover those applicable to multistage low-pressure gas turbines as identified by Halstead et al.<sup>24</sup> Worst-case experimental uncertainties for  $Tu_\infty$  and  $Re_\theta$  were 3.5 and 7%, respectively.

#### Boundary-Layer Response to FST

Typical velocity profiles with FST are given in Fig. 7. For comparison to the CFS validation cases, the DNS results of Spalart<sup>32</sup> are shown. The mean profile shows strong agreement with the DNS through the buffer layer and a portion of the log layer.

Measured  $C_f$  are shown in Fig. 8. When compared to Eq. (6), the CFS  $Re_\theta$ -based correlation, enhancements between 6 and 16% are observed in the present data.

### Analysis of Results

#### Established Skin-Friction Comparisons

Figure 9 shows measured  $C_f$  enhancements in terms of  $HB_{mod}$ . To facilitate consistency and to accurately represent the correlative scatter a design engineer might encounter, Eq. (6) is used as the CFS reference for all data sets; actual  $C_f$  values were compared to those predicted using Eq. (6) without adjustment for a specific experimental apparatus. The  $HB_{mod}$  parameter collects  $\Delta C_f/C_{f0}$  to within about  $\pm 40\%$ . When the functional dependence indicated by the solid line in Fig. 9 is used, a variation of  $\pm 4\%$  ( $\pm 2\sigma$  of data shown) is found for  $(C_f/2)^{1/2}$  predictions using the  $HB_{mod}$  parameter; direct  $(C_f/2)^{1/2}$  results are shown in Fig. 10. [We choose to

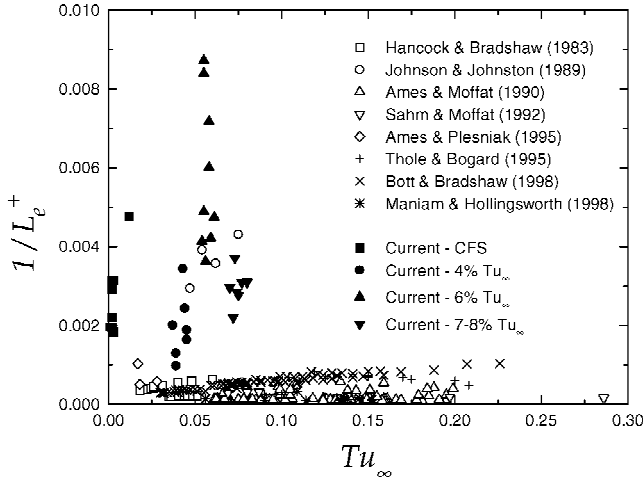


Fig. 6 Length scale ratios in current and previous studies ( $1/L_e^+$  vs  $Tu_\infty$ ).

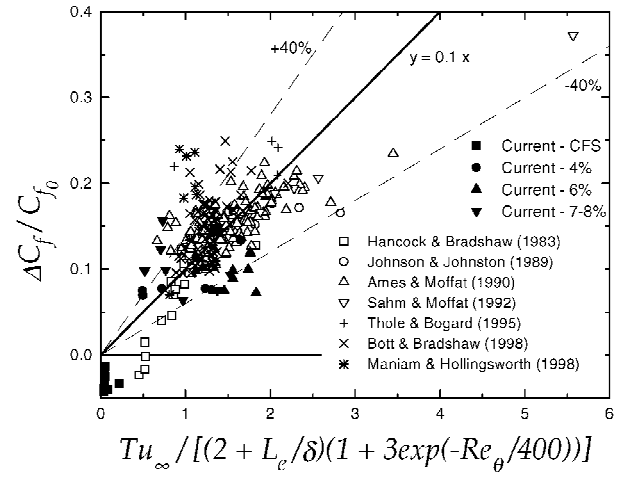


Fig. 9  $C_f$  enhancement vs modified HB.

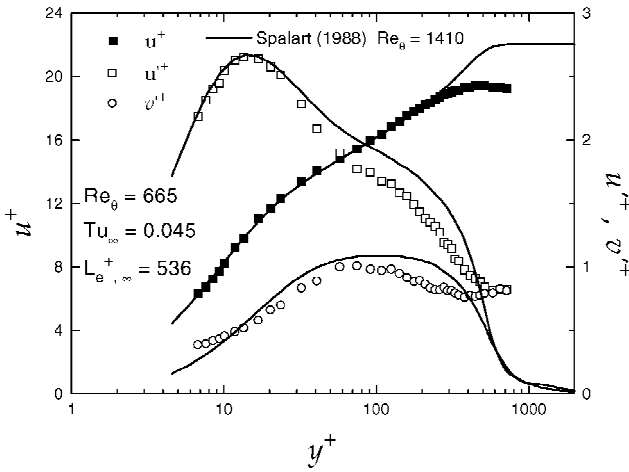


Fig. 7 Dimensionless velocity profiles with elevated  $Tu_\infty$ .

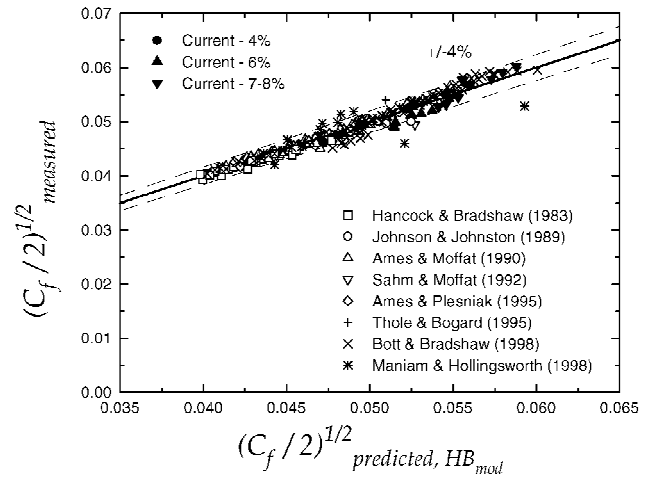


Fig. 10 Measured  $C_f$  vs predicted  $C_f$  using modified HB.

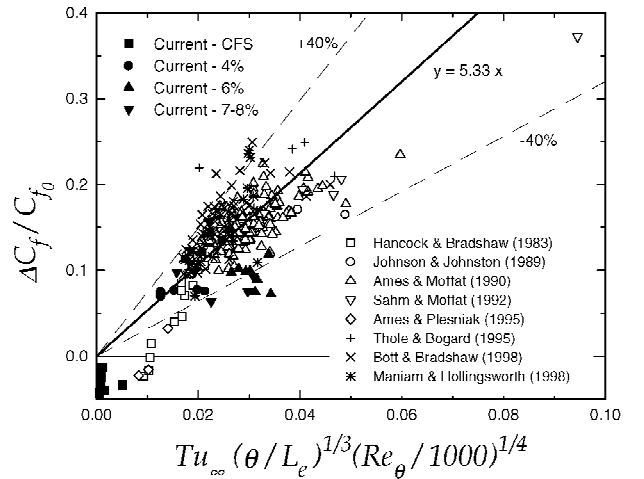


Fig. 11  $C_f$  enhancement vs TLR<sub>m</sub>.

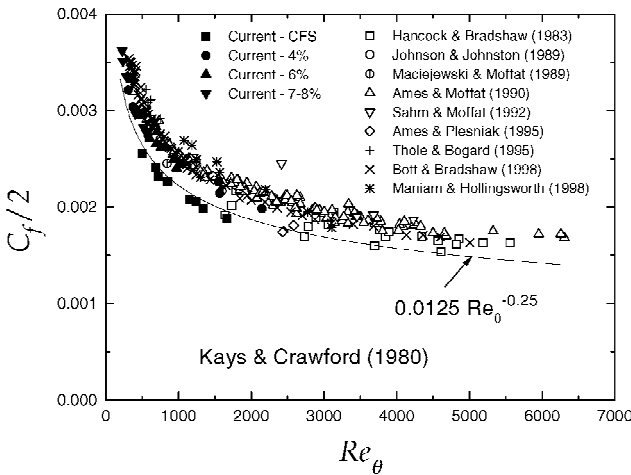


Fig. 8 Skin-friction coefficient vs  $Re_\theta$ .

display the data as  $(C_f/2)^{1/2}$ , that is,  $u_\tau/\bar{u}_\infty$ , because this velocity ratio can be used to correlate heat transfer using only freestream parameters.<sup>30,33]</sup> Figure 11 shows that TLR<sub>m</sub> behaves similarly when examining  $C_f$  enhancement. Presented in Fig. 12, TLR<sub>m</sub> yields direct  $(C_f/2)^{1/2}$  predictions to within  $\pm 3\%$ . The collapse of data using these two  $Re_\theta$ -based correlations is quite remarkable when one considers that first-order experimental uncertainty in  $C_f$  typically ranges from  $\pm 3$  to  $\pm 6\%$ . The equivalent first-order

uncertainty in  $(C_f/2)^{1/2}$  of  $\pm 1.5\%$  to  $\pm 3\%$  leaves little for  $n$ th-order effects.

Thole and Bogard<sup>15</sup> observed variation in the value of  $C'_f$  due to its dependence on  $Tu_\infty$ . Combining the current small-scale data with earlier observations in Fig. 13, an explicit dependence on  $L_e/\delta$  is now also apparent. Over the length scale range shown, the value of  $C'_f$  drops approximately 50%. A log-linear regression analysis yields odds of less than  $1:10^{19}$  ( $p < 10^{-19}$ ) that  $C'_f$  and  $L_e^+$  are not correlated.



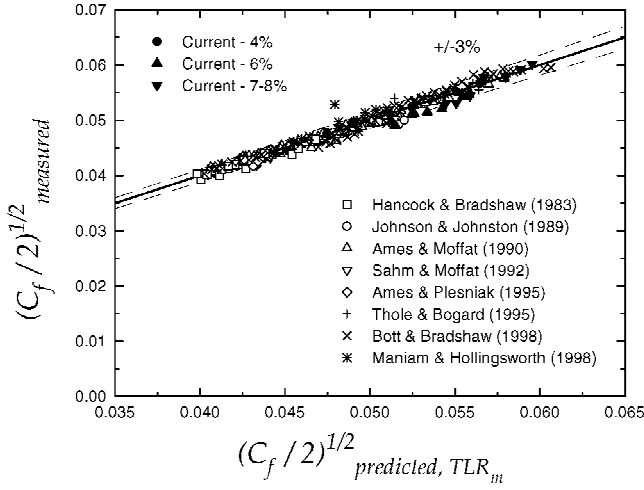


Fig. 12 Measured  $C_f$  vs predicted  $C_f$  using  $TLR_m$ .

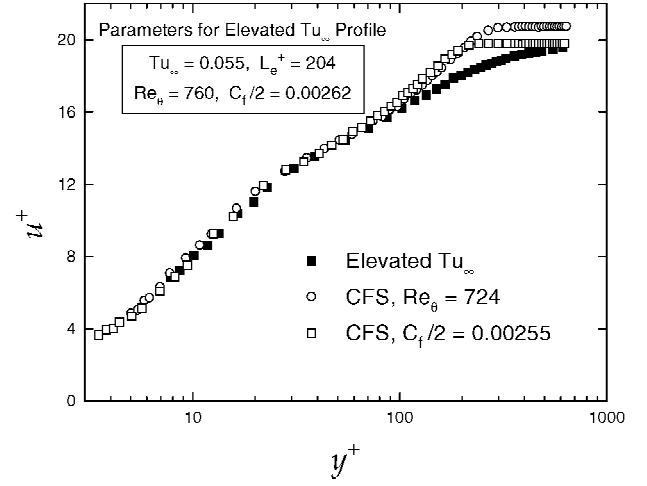


Fig. 14 Elevated  $Tu_{\infty}$  and CFS profile comparison.

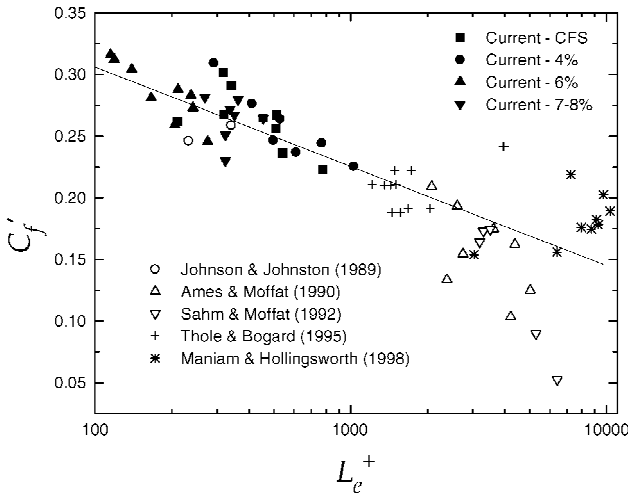


Fig. 13  $C'_f$  dependence on length scale.

#### Development of $C_f$ Correlation

When the effect of FST is characterized in terms of  $\Delta C_f/C_{f0}$ , the need for advance knowledge of  $Re_{\theta}$  limits the predictive usefulness of the correlation. Our data indicate, as other investigators have shown, that, when streamwise pressure gradients are small, the  $u^+$  profile in the sublayer and the buffer layer is largely unchanged regardless of  $Tu_{\infty}$ . Often, a significant portion of the log-layer also follows Eq. (5). To develop a  $C_f$  correlation that does not require  $Re_{\theta}$  information, we begin by examining the two-dimensional boundary-layer momentum equation and the implications of a universal  $u^+$  profile in the sublayer and buffer layer. The two-dimensional Reynolds averaged  $x$ -momentum equation for an incompressible Newtonian fluid with negligible bulk viscosity and constant properties is given by

$$\bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} = \frac{\partial}{\partial y} \left( \bar{v} \frac{\partial \bar{u}}{\partial y} - \overline{u'v'} \right) \quad (7)$$

Equation (7) assumes the body forces and streamwise pressure gradient are negligible. Making the equation dimensionless using wall variables, we write

$$u^+ \frac{v}{u_{\tau}^2} \frac{\partial \bar{u}}{\partial x} + v^+ \frac{\partial u^+}{\partial y^+} = \frac{\partial}{\partial y^+} \left( \frac{\partial u^+}{\partial y^+} - uv^+ \right) \quad (8)$$

When the incompressible continuity equation is employed, Eq. (8) can be written

$$-u^+ \frac{\partial v^+}{\partial y^+} + v^+ \frac{\partial u^+}{\partial y^+} = \frac{\partial}{\partial y^+} \left( \frac{\partial u^+}{\partial y^+} - uv^+ \right) \quad (9)$$

We temporarily focus on only the sublayer and buffer layer; let us assume that  $u^+ = u^+\{y^+\}$  is universal as the data<sup>34</sup> suggest. Equation (9) then directly governs the relationship between  $v^+$  and  $uv^+$  near the wall. If  $v^+$  were dependent on  $y^+$  only, then Eq. (9) would be an ordinary differential equation with  $y^+$  as the independent variable. For an impenetrable surface, the inner boundary conditions for  $u^+$ ,  $v^+$ , and  $uv^+$  all equal zero. Although there are still two dependent variables in the equation and we recognize that the solution is not unique, Eq. (9) demonstrates that near-wall behavior is tightly restricted by the value of  $uv^+$  at the buffer-layer outer boundary (the inner edge of the boundary-layer core). Additional constraints related to likely monotonic behaviors in this region further limit the set of possible solutions. This entire approach suggests that viewing the mean velocity profile in terms of a near-wall and a core region may prove useful.

Focus again on the entire boundary layer: Figure 14 shows three  $u^+$  profiles. One profile is for a flow with FST; the other two are CFS profiles, one at nearly the same  $Re_{\theta}$  and one with nearly the same  $C_f$  as that for the FST case. The like- $Re_{\theta}$  profiles have little in common outside a  $y^+$  value of 100. The like- $C_f$  profiles, however, also share outer boundary conditions for  $u^+$  [ $u_{\infty}^+ = (C_f/2)^{-1/2}$ ] and  $uv^+$  ( $uv_{\infty, CFS}^+ = 0$  and  $uv_{\infty, isotropic FST}^+ = 0$ ).

The desire to obtain similar outer boundary conditions dictates that we compare profiles with similar  $C_f$  values. Recalling that Eq. (9) governs both of these flows and expecting that momentum transport in the core of the boundary layer is dominated by the turbulent component, we propose that the differences in the like- $C_f$  profiles are due primarily to the characteristics of the turbulence in the boundary-layer core. If a gradient-diffusion model that characterizes the diffusivity using a single velocity and length scale can effectively represent the turbulent transport in the core, this proposal leads to the functional statement

$$C_f = C_f(\mathbf{V}, \mathbf{L}) \quad (10)$$

where  $\mathbf{V}$  and  $\mathbf{L}$  are the respective characteristic velocity and length scales. We now develop a  $C_f$  correlation in accord with this result.

Maniam and Hollingsworth<sup>13</sup> found that, for the CFS turbulent boundary layer,

$$u'_{15}/u_{\tau} = 2.06 Re_{\theta}^{1/24} \quad (11)$$

Using CFS data taken at  $y^+ = 150$ , Barrett<sup>29</sup> developed the relationship

$$L_e^+ = 2.25 Re_\theta^{\frac{3}{4}} \quad (12)$$

relating  $L_e^+$  in the core of the boundary layer to  $Re_\theta$ . Combining Eqs. (11) and (12), we write

$$u'_{15}/u_\tau = 1.97(L_e^+)^{1/18} \quad (13)$$

Figure 15 shows a comparison of Eq. (13) with data from this and other studies regardless of FST level. Freestream values were used for  $L_e$  in all cases with FST; values at  $y^+ = 150$  were used for the CFS cases. When it is considered that Eq. (13) was derived using only CFS information, except for one high  $Tu_\infty$  case of Sahm and Moffat,<sup>11</sup> the data in Fig. 15 are encouraging.

We develop the correlation by first writing

$$\sqrt{C_f/2} = u_\tau/\bar{u}_\infty = (u_\tau/u'_{15})(u'_{15}/u'_\infty)(u'_\infty/\bar{u}_\infty) \quad (14)$$

and then rewriting using Eq. (13) and the definition of  $Tu_\infty$ :

$$\sqrt{C_f/2} = (1/1.97)(L_e^+)^{-1/18}(u'_{15}/u'_\infty)Tu_\infty \quad (15)$$

We search for a single dimensionless velocity and length scale combination to correlate  $C_f$ . If  $u'_{15}/u'_\infty$  is solely a function of  $Tu_\infty$  and  $L_e^+$ , then these two scales will satisfy our criteria. A set of  $u_\tau^+$  profiles is shown in Fig. 16. Note the relationship between  $u'_{15}$ ,  $u_\tau^+$ , and

$Tu_\infty$ . The potential of an identifiable relationship based on this observation motivates our examination of the functional dependence between  $u'_{15}/u'_\infty$  and  $Tu_\infty$ . To extract data from studies that did not measure  $u'_{15}$ , we chose to calculate the  $u'_{15}/u'_\infty$  value by rearranging Eq. (15):

$$u'_{15}/u'_\infty = 1.97\sqrt{C_f/2}(L_e^+)^{1/18}/Tu_\infty \quad (16)$$

The result using nine data sets is shown in Fig. 17. For consistency, Eq. (16) was used to analyze the data from each study. Despite the wide range of flow conditions represented, all of the data in Fig. 17 are well represented by the power law fit given by

$$u'_{15}/u'_\infty = 0.231 Tu_\infty^{-0.82} \quad (17)$$

We now combine Eqs. (15) and (17) to arrive at the  $C_f$  correlation

$$\sqrt{C_f/2} = TL^+ \quad (18)$$

where  $TL^+$  is defined by

$$TL^+ \equiv 0.117(L_e^+)^{-1/18} Tu_\infty^{0.18} \quad (19)$$

Figure 18 shows a comparison of Eq. (18) with the data from numerous authors. For a  $u_\tau/\bar{u}_\infty$  prediction in terms of only  $Tu_\infty$  and  $L_e^+$  (not  $Re_\theta$ ), the agreement is again encouraging. Equation (18) provides  $(C_f/2)^{1/2}$  predictions to within  $\pm 8\%$  ( $\pm 2\sigma$  of data shown). The data shown in Fig. 18 cover  $Tu_\infty$  from 2 to 30%,  $L_e^+$  from 100 to 11,000, and  $Re_\theta$  from 300 to 6300. To predict  $(C_f/2)^{1/2}$  using Eq. (18), we need know only

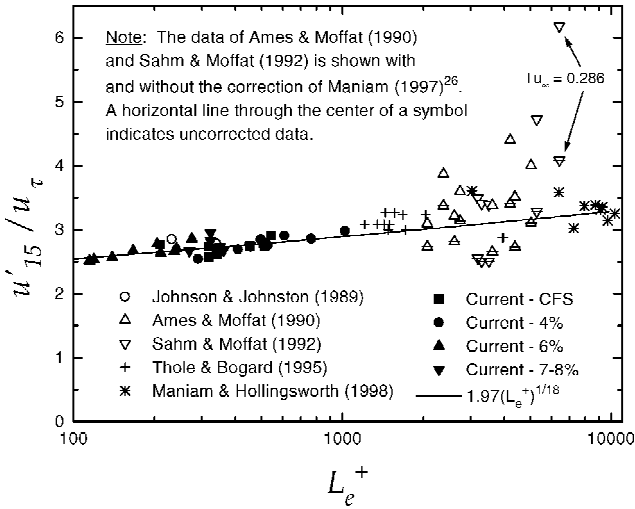


Fig. 15 Near-wall fluctuation vs length scale.

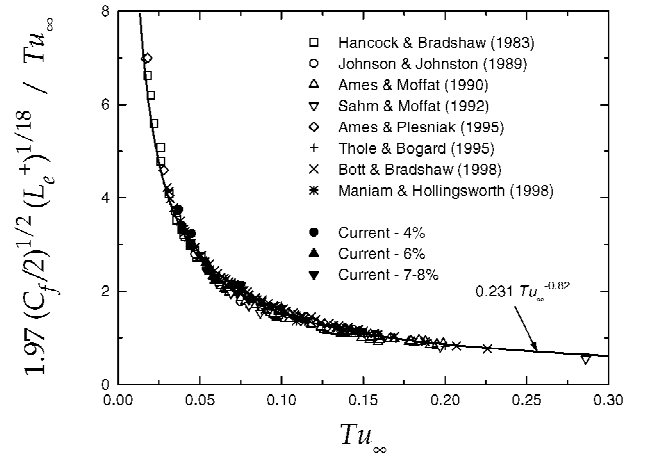


Fig. 17 Velocity-ratio relationship to  $Tu_\infty$ .

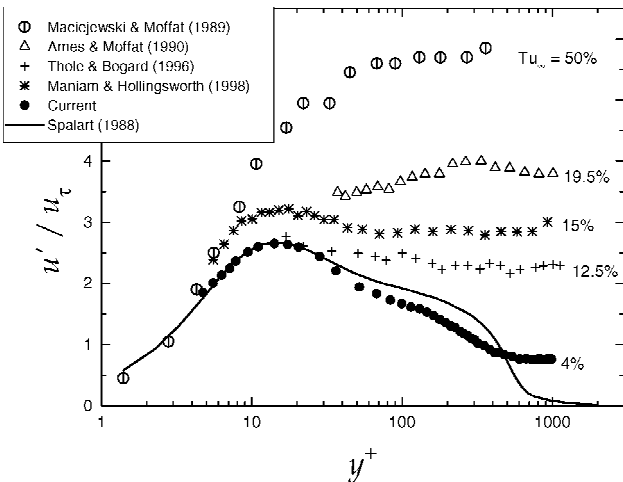


Fig. 16 Profiles of  $u_\tau^+$  from various studies at different levels of  $Tu_\infty$ .

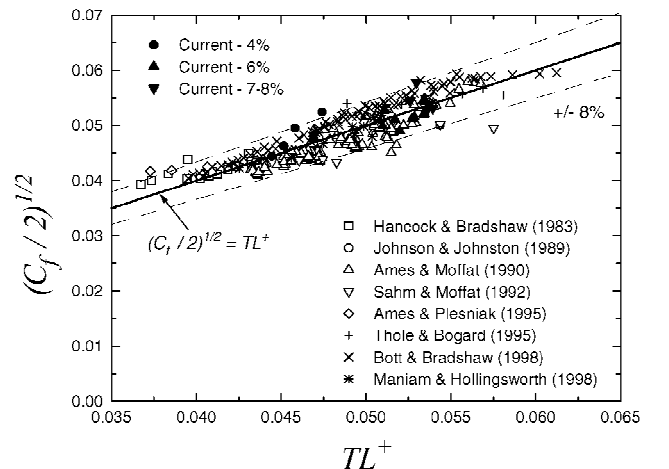
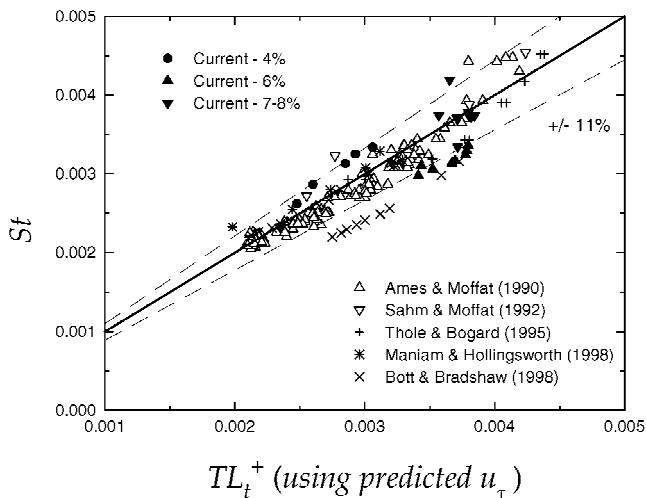


Fig. 18 Dimensionless friction velocity correlated using  $TL^+$ .

**Table 1 Summary of scatter and input information for the various correlations**

Parameter	HB <sub>mod</sub>	TLR <sub>m</sub>	TL <sup>+</sup> [Eq. (19)]
±2σ Departure on (C <sub>f</sub> /2) <sup>1/2</sup> , %	4	3	8
Tu <sub>∞</sub>	Required	Required	Required
L <sub>e,∞</sub>	Required	Required	Required
ū <sub>∞</sub>	Required	Required	Required
θ	Required	Required	



**Fig. 19 Heat transfer correlation using Eq. (18).**

$\bar{u}_{\infty}$ ,  $u'_{\infty}$ ,  $L_{e,\infty}$ , and the fluid properties. When these parameters are used, the unknown in the equation,  $u_{\tau}$ , is determined and  $C_f$  is calculated.

To evaluate its performance, we compare  $TL^+$  to earlier correlations. Table 1 summarizes the scatter and information requirements for the aforementioned methods. When scatter of  $\pm 8\%$  is accepted, Eq. (18) enables a heat transfer correlation<sup>30,33</sup> (also using only freestream information) to capture Stanton number  $St$  to within  $\pm 11\%$ . Data illustrating this correlation are shown in Fig. 19.

## Conclusions

Very small-scale, moderate-intensity FST enhances turbulent boundary-layer skin friction up to 16% higher than that predicted using an accepted CFS Reynolds number-based correlation. The HB<sub>mod</sub> and TLR<sub>m</sub> elevated-turbulence correlations adequately capture the small-scale data set.

The  $C'_f$  coefficient correlates with freestream length scale. Over the length scale range examined,  $C'_f$  drops approximately 50%. A log-linear regression analysis confirms that  $C'_f$  and  $L_e$  are strongly correlated ( $p < 10^{-19}$ ). The new small-scale data make this observation possible.

For flat-plate turbulent boundary layers subjected to FST, the new  $TL^+$  correlation is able to predict the dimensionless friction velocity using only freestream information. Combining the freestream length scale with mean and fluctuating freestream velocities provides enough information to estimate the dimensionless friction velocity to within  $\pm 8\%$ . The ratio of turbulent-length-scale-to-viscous-length-scale  $L_e^+$  is used as the characteristic dimensionless length scale.

The new  $TL^+$  correlation compares moderately well in accuracy to earlier correlation methods. The 8% uncertainty band for the new correlation is roughly twice as large as the earlier correlations; however, both HB<sub>mod</sub> and TLR<sub>m</sub> require a priori knowledge of momentum-thickness Reynolds number. Because the  $TL^+$  method does not require  $Re_{\theta}$  information, over a convincingly large range of flow conditions, the new correlation is a viable predictive tool. Modifications to include pressure gradient, curvature, and other complicating effects should be pursued.

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